

Clock Synchronisation in the Vicinity of the Earth

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Abstract

The transmission time of an electromagnetic signal in the vicinity of the earth is calculated to c^{-2} and contains an orbital Sagnac term. On earth, the synchronisation of *TCB* can be realised by atomic clocks, but not the one of *TCG*. The principle of equivalence is discussed.

Keywords: relativity, Sagnac effect, synchronisation, time scale.

1 Introduction

In 1994, Petit and Wolf [1] treated the problem of light propagation with the goal of ensuring picosecond accuracy for time transfer techniques using electromagnetic signals in the vicinity of the earth. They showed that the first post-Newtonian approximation of the geocentered metric, as defined by the resolution A4 of the International Astronomical Union (IAU) [2], was sufficient for their purpose and calculated the one-way time transfer of light to be applied when the spatial coordinates of the time transfer stations are known in a geocentric reference frame rotating with the earth. This expression was used to calculate at the same accuracy the special cases of the two-way and LASSO time transfers via geostationary satellites. To the required accuracy, within a geocentric sphere of 200'000 km, the transmission delay is the sum of two terms: a geometrical one and a gravitational one. The geometrical one contains, under other, the Sagnac effect [3] due to the rotation of the earth on itself.

The aim of this article is to show that another effect arises in the one-way transmission time at the required accuracy: the Sagnac effect due to the orbital motion of the earth. In section 2, we present the reasons why such an effect takes place, the

theoretical implications on the principle of equivalence as well as on the choice of a coordinate time scale on earth. In section 3, we calculate it explicitly in the same way as Petit and Wolf did in their article. In section 4, we discuss the implications on the realisability of the resolution A4 of the IAU.

2 Orbital Sagnac effect

The reason why Petit and Wolf did not take the orbital Sagnac effect into account is the following [4]: “ I think (P. Wolf too) that what would be true in special relativity (i.e. if the reference frame bound to the earth would have for a reason X a displacement similar to the earth orbit and thus would be accelerated) is no more true in general relativity, where the earth is in free fall and only tidal effects subsist, which we have evaluated and found < 1 ps in our paper.” We think that this point of view is in contradiction with some experiments for the reasons expounded in the following paragraphs.

The Sagnac effect was observed around the earth using GPS satellites in common-view between 3 pairs of timing centers: Boulder, Tokyo and Braunschweig [5]. Let us call L the length covered by the light signal in this experiment. Saying that an around-the-world Sagnac effect occurs is just like saying that the time that light needs to do the round trip Boulder - Satellite 1 - Braunschweig - Satellite 2 - Tokyo - Satellite 3 - Boulder is not given by L/c when measured in the rotating (relative to distant radio sources) frame of the earth, but a Sagnac “correction” has to be applied and is equal in second order to $2\omega A_E/c^2$, where A_E is the sum of the areas of the equatorial projections of the triangles whose vertices are the center of the earth, the position of a timing center and a satellite and ω is the angular velocity of the earth. A_E is positive for signal propagation in the eastward direction and negative otherwise.

In order to prove that a Sagnac effect occurs also in free fall, we can imagine a slightly different experiment, where light signals are sent around the world between 3 satellites (the first one being geostationary), which are in positions such that the timing centers are on the straight line between two satellites. Light makes the round trip: Satellite 1 - Timing center 1 - Satellite 2 - Timing center 2 - Satellite 3 - Timing center 3 - Satellite 1. Just as in the case of the real experiment, a Sagnac effect will be measured also in this particular configuration, but in this case the timing centers play no role, since light could go directly from one satellite to the other, and we conclude that a Sagnac effect takes also place between the satellites only, in spite of the fact that they are in free fall. This conclusion is also valid if two satellites are so close to each other inside a region where it is possible to choose locally a freely falling inertial frame. In post-Aristotelian physics, there is no reason to think, that there are different physical laws for satellites around the earth and the earth around the sun.

So, is that not true that on earth, because we are in free fall, we remain with tidal effects only? In our opinion, this is a widespread, but abusive interpretation of the principle of equivalence in general relativity. This last states in fact: “ At every space-time point in an arbitrary gravitational field it is possible to choose a “locally inertial

coordinate system” such that, within a sufficiently small *four-dimensional* region of the point in question, the laws of nature take the same form as in unaccelerated Cartesian coordinate systems in the absence of gravitation”. The four-dimensionality of the neighbourhood is essential in this formulation [6, p. 17], and referring to the earth as a region with negligible mass, we can only say that we are left with tidal effects only in a sufficiently small three-dimensional region, during a sufficiently small time interval, but certainly not during a whole revolution of the earth around the sun.

Let us now consider the earth as a point of negligible mass rotating in circle around the sun and let us imagine an infinity of mirror-satellites on the earth orbit, maintaining a constant distance between them. These mirror-satellites would able us to send a light ray from earth, which would be constrained to follow the circular orbit and come back on earth. The reader familiar with the Sagnac effect, will recognize the simplified configuration used for pedagogical purposes. Using a barycentric reference system (*BRS*) centered on the sun and non-rotating respective to the distant radio sources, we calculate easily, that a light ray sent on the orbit will come back after a time delay Δt given by:

$$\Delta t = \frac{L}{c} + \frac{2A_S\omega}{c^2} + O(c^{-3}) = \frac{L}{c} \pm \frac{Lv_E}{c^2} + O(c^{-3}) \quad , \quad (1)$$

where L is the length of the earth orbit, A_S is the oriented area of the earth orbit, ω the rotation vector of the earth around the sun and v_E is the velocity of the earth. From (1), we find that the velocity of light around its orbit is not c , but in first approximation $c \mp v_E$, when measured on earth. Following the principle of equivalence, we can consider that in every point of the orbit at the moment of the passage of the light ray, there is a four-dimensional local coordinate system going at the velocity of the mirror. Then, in every of this frame the special relativity theory (SRT) is valid and in particular the velocity of light is c . This means that a light ray would take the time $\Delta t = L/c$ to make a round trip and negate the Sagnac effect.

The problem above is nothing else than the transposition of the problem of the rotating platform from flat space-time to the curved space-time of general relativity. As already stressed by Selleri [7](see also [8, 9]), SRT applied locally on every small tangential part of a rotating platform is unable to explain the Sagnac effect on this platform. Langevin considered also the problem long ago [10]. He gave two different explanations of the Sagnac effect on the rotating platform. The first one is to consider a global central time such as the time of the laboratory. Time is then defined everywhere and self-consistently but such a synchronisation of clocks gives a velocity of light on the rim of the platform which is not constant but in first approximation $c - \omega r$ and $c + \omega r$, where ω is the angular velocity of the platform and r its radius. The second “solution” is to consider that the velocity of light is c on the rim of the platform. This is equivalent to choose a local time, which is given by integration of the differential of the global time plus a differential contribution expressed in the space variables. The problem is that this contribution is not a total differential. Thus, the synchronisation procedure is path dependent and time cannot be defined consistently.

This problem has been recognized on earth, by members of the timing community [11] : “This means that Einstein synchronization in a rotating reference frame is not self-consistent.[...]. In order to avoid difficulties with such non-transitivity it is best to adopt time in the *non-rotating* frame as the measure of time in the rotating frame.” It can be easily proved that this problem occurs every time that the metric is non-static [12].

Despite the fact that the metric of the solar system is static, the problem is similar to the rotating platform. The global time in the solar system is the Barycentric Coordinate Time (*TCB*) [2]. Let us consider two clocks at time *TCB* located at \mathbf{x} and $\mathbf{x} + \mathbf{dx}$ and rotating with velocity \mathbf{v}_E on a circular orbit around the sun which are synchronised by adopting the time of the non-rotating frame. Then, the one-way velocity of light between the two clocks will be in first approximation $c \pm v_E$. If we use a local time, that is the Geocentric Coordinate Time (*TCG*), the velocity of light will be c in first approximation. Comparing two *TCB*-synchronous events in points \mathbf{x} and $\mathbf{x} + \mathbf{dx}$, we see that they are not *TCG*-synchronous, but we have, using the transformation given in [2]:

$$\begin{aligned} TCB(\mathbf{x}) - TCG(\mathbf{x}) &= \frac{1}{c^2} \int_{TCB_0}^{TCB} (v_E^2/2 + U_{ext}(\mathbf{x})) dt \\ TCB(\mathbf{x} + \mathbf{dx}) - TCG(\mathbf{x} + \mathbf{dx}) &= \frac{1}{c^2} \left[\int_{TCB_0}^{TCB} (v_E^2/2 + U_{ext}(\mathbf{x} + \mathbf{dx})) dt \right. \\ &\quad \left. + \mathbf{v}_E \cdot \mathbf{dx} \right] , \end{aligned} \tag{2}$$

so that:

$$\Delta TCG - \Delta TCB = \Delta TCG \cong -\mathbf{v}_E \cdot \mathbf{dx}/c^2 , \tag{3}$$

where the geocentric reference system (*GRS*) is centered on \mathbf{x} for simplicity, Δ expresses the difference of time between the two points and from (2) to (3) the difference of the external potential U_{ext} between \mathbf{x} and $\mathbf{x} + \mathbf{dx}$ can be neglected. Trying to extend *TCG* spatially out of a local domain, for example by synchronising clocks along the orbit, we see that the procedure is path dependent, because $\mathbf{v}_E \cdot \mathbf{dx} = \omega r^2 d\theta$ is not a total differential in r and θ , where r and θ are the polar coordinates of \mathbf{x} in *BRS*. If we now try to extend *TCG* temporally, we can imagine two clocks, fixed in *GRS*, that is which have a constant direction respective to distant radio sources. Applying two times (3) (that is to *GRS* at the point $\mathbf{x}(t)$ and later at point $\mathbf{x}(t' > t)$) to two pairs of *TCB*-simultaneous events, we see that the simultaneity in the two successive *GRS* frames is not the same. So, *TCG*-synchronised clocks in the first *GRS* frame would have to be resynchronised in order to obey to the different simultaneity of the second *GRS*. We will consider again this problem in section 4. So *TCG* has highly undesirable properties and it is better to consider, on the moving earth, that the simultaneity is given by the coordinate synchronisation of *TCB*.

This solution, which is the only one allowing a global and self-consistent definition of time, is also allowed by the principle of equivalence, which only states that it is *possible* to choose a local coordinate system such that the law of SRT are valid, but does not avoid to make an other choice such that the velocity of light is not

invariant. More precisely: If in any point of a four dimensional surface, with general coordinates x^α , ($\alpha = 0, 1, 2, 3$), there is a local transformation $x \rightarrow \xi$ such that the metric is locally Minkowskian, we can do a further resynchronisation of clocks $\xi^0 \rightarrow \xi'^0(\xi^0, \xi^i)$ and $\xi^i \rightarrow \xi'^i = \xi^i$, ($i = 1, 2, 3$). In particular it is possible to choose a transformation such that the time is globally defined on the whole surface, because x^0 is already global. If the coordinates of the velocity of the unprimed system are written in a “vector” form: $\mathbf{v} = \frac{d\mathbf{x}}{dt}$, this transformation is given by: $\xi^0 \rightarrow \xi'^0 = \xi^0 + \mathbf{v} \cdot \boldsymbol{\xi}/c$, where $\boldsymbol{\xi} = (\xi^1, \xi^2, \xi^3)$. The laws of physics in the primed (local) system are the laws of the inertial theory [7], a theory equivalent to special relativity in inertial systems but maintaining an “absolute” simultaneity [13]. The metric expressed in the primed local coordinates is given by [14]:

$$g_{\alpha\beta} = \begin{pmatrix} -1 & v_1/c & v_2/c & v_3/c \\ v_1/c & 1 - v_1^2/c^2 & -v_1v_2/c^2 & -v_1v_3/c^2 \\ v_2/c & -v_1v_2/c^2 & 1 - v_2^2/c^2 & -v_2v_3/c^2 \\ v_3/c & -v_1v_3/c^2 & -v_2v_3/c^2 & 1 - v_3^2/c^2 \end{pmatrix} \quad (4)$$

The spatial metric γ_{ij} ($i, j = 1, 2, 3$), which may be calculated from $\gamma_{ij} = g_{ij} - g_{0i}g_{0j}/g_{00}$ is equal to $diag(1, 1, 1)$, as one can calculate from (4).

3 One-way transmission time

Let us now calculate the one-way transmission time of an electromagnetic signal between two points a and b , given by their coordinates \mathbf{x}_{ra} and \mathbf{x}_{rb} in the geocentric rotating frame of the earth. Following Petit and Wolf, we calculate it in the non-rotating frame (coordinates: $\mathbf{x}_a, \mathbf{x}_b$), where the path is a straight line, but with the clock synchronisation of TCB , which allows a global definition of time. We suppose that all clocks are rate corrected for their velocities and for the gravitational potential at their position so that they run at the rate of TT . One can show [1], that up to an accuracy of order $O(c^{-3})$, the transmission time T_t , can be written as the sum of a geometrical (T) and a gravitational term (T_g). That is: $T_t = T + T_g$. T_g is of order c^{-3} and the choice of the coordinate synchronisation of TCB rather than the one of TCG , introduces corrections that are negligible (of order c^{-5}), so that T_g is given by (14) of [1]. Afterwards we will consider only the geometry and limit ourself to a calculation of order c^{-2} , since differences with [1] appear already at this order of approximation. An electromagnetic signal is sent at time t_0 from a and arrives at time t_1 in b . From (4), writing: $ds^2 = g_{\alpha\beta}dx^\alpha dx^\beta = 0$ and solving the resulting quadratic equation in dx^i/dt ($i = 1, 2, 3$), one obtains the velocity of light in a direction $\hat{\mathbf{n}}$:

$$\mathbf{c}(\hat{\mathbf{n}}) = \frac{c\hat{\mathbf{n}}}{1 + \frac{\mathbf{v}_E \cdot \hat{\mathbf{n}}}{c}} = \hat{\mathbf{n}}(c - \mathbf{v}_E \cdot \hat{\mathbf{n}} + O(c^{-1})) \quad , \quad (5)$$

where \mathbf{v}_E is the velocity of the earth in BRS . The transmission time is given by:

$$T = (1 - U_g/c^2) | \mathbf{x}_{rb}(t_0) - \mathbf{x}_{ra}(t_0) | /c(\hat{\mathbf{n}}) + s \quad , \quad (6)$$

where s represents the time taken by the signal to traverse the extrapath due to the motion of b in the non-rotating frame during the transmission, and the factor $(1 - U_g/c^2)$ with $U_g/c^2 = L_g$ arises because T is measured in units of TT (see [2]: $L_g = 6.969291 \cdot 10^{-10}$). Defining:

$$\begin{aligned}\mathbf{R}_0 &= \mathbf{x}_{\mathbf{rb}}(t_0) - \mathbf{x}_{\mathbf{ra}}(t_0) \\ \mathbf{v}_b &= \boldsymbol{\omega} \times \mathbf{x}_{\mathbf{rb}} + \mathbf{v}_{\mathbf{rb}} + O(c^{-2}) ,\end{aligned}\tag{7}$$

with $\mathbf{v}_{\mathbf{rb}}$ being the velocity of b in the rotating frame and the two frames having their axes pointing in the same direction at $t = t_0$.

The path travelled by the signal $\mathbf{R}(T)$ can be expressed as a series expansion in terms of T in the non-rotating frame:

$$\mathbf{R}(T) = \mathbf{R}_0 + \mathbf{v}_b T + O(T^2)\tag{8}$$

Calculating its magnitude $R(T)$, expanding the square root and writing $R(T) = c(\hat{\mathbf{n}})T/(1 - U_g/c^2)$ we find:

$$T = [R_0 + (\mathbf{R}_0 \cdot \mathbf{v}_b/R_0)T + O(T^2)](1 - U_g/c^2)/c(\hat{\mathbf{n}})\tag{9}$$

Starting with $T = R_0/c(\hat{\mathbf{n}})$ and iterating once yields an expression for the transmission time in terms of the known quantities \mathbf{R}_0 and \mathbf{v}_b :

$$T = R_0/c + \mathbf{R}_0 \cdot (\mathbf{v}_E + \mathbf{v}_b)/c^2 + O(c^{-3})\tag{10}$$

and Petit and Wolf found at second order in c :

$$T^{TCG} = R_0/c + \mathbf{R}_0 \cdot \mathbf{v}_b/c^2 + O(c^{-3})\tag{11}$$

Strictly speaking, the transmission time calculated in (10) does not correspond to the same physical situation as the transmission time calculated in (11) The reason is the following: \mathbf{R}_0 is not the same in both situations since it depends on the chosen synchronisation. In theory, the length between two points is measured with unit rods between the simultaneous positions of these two points. If these points are fixed in a given reference frame the notion of simultaneity does not influence the result of a measure in this frame [15]. But if the points are moving, this is no more true because the same numerical value of the coordinate time in different synchronisations does not correspond to the same position of the moving objects and inversely the points are at the same places at different times in different synchronisations. From $t^{TCB} \cong t^{TCG} + \mathbf{v}_E \cdot \mathbf{x}/c^2$ and (7), we calculate easily:

$$\begin{aligned}\mathbf{R}_0^{TCB} &= \mathbf{R}_0^{TCG} + c^{-2} \left[\mathbf{v}_{\mathbf{rb}}^{TCG} (\mathbf{v}_E \cdot \mathbf{x}_{\mathbf{rb}}^{TCG}(t_0)) \right. \\ &\quad \left. - \mathbf{v}_{\mathbf{ra}}^{TCG} (\mathbf{v}_E \cdot \mathbf{x}_{\mathbf{ra}}^{TCG}(t_0)) \right] + O(c^{-4})\end{aligned}\tag{12}$$

The velocities are also not the same and vary at second order.

$$\mathbf{v}_b^{TCB} = \mathbf{v}_b^{TCG} (1 - \mathbf{v}_E \cdot \mathbf{v}_b^{TCG}/c^2) + O(c^{-4})\tag{13}$$

Substituting \mathbf{R}_0 and \mathbf{v}_b of (10) by their value of (12) and (13) does not change the result at this order of approximation. So, we can say that (10) and (11) are calculated in the same physical situation, at $O(c^{-3})$, but with different synchronisations. The additional term in (10) respective to (11) comes from (5) and is the orbital Sagnac effect. It is given in first approximation by $\mathbf{R}_0 \cdot \mathbf{v}_E/c^2$ and does not occur if \mathbf{R}_0 is perpendicular to the orbital velocity of the earth. If \mathbf{R}_0 is parallel to \mathbf{v}_E it amounts 333 ns for a link of 1000 km. Of course the presence this new term respective to the expression of Petit and Wolf is only measurable if we adopt the synchronisation of *TCB*. The question is then: will such an orbital effect arise if we do not adopt this “convention”?

4 Is the synchronisation of TCG realisable on earth

If we use the coordinate synchronisation of *TCG* on earth as recommended by the IAU, the orbital Sagnac effect will not be measured immediatly, but a desynchronisation of clocks will occur. This is due to the fact that two clocks *A* and *B*, located in \mathbf{x}_A and \mathbf{x}_B , which are Einstein synchronised in an inertial frame do not remain Einstein-synchronous after or during acceleration [16]. For the same reasons, that are expounded in section 2, this fact is not only true within the frame of special relativity but also for two freely falling clocks, close to each other and in orbit around the sun. The desynchronisation of clocks is given by [11]:

$$\Delta t = t_B - t_A = -\frac{\mathbf{u} \cdot \mathbf{R}}{c^2} , \quad (14)$$

where \mathbf{u} is the relative velocity of the two frames and $\mathbf{R} = \mathbf{x}_B - \mathbf{x}_A$ is the vector separating the two clocks. So if realise *TCG* at time $t_0 = 0$ and we call the corresponding non-rotating frame *GRS*₀, then the velocity \mathbf{u} of the earth relative to *GRS*₀ will be for a circular orbit:

$$\mathbf{u} = v_E \begin{pmatrix} -\sin \theta \\ \cos \theta - 1 \end{pmatrix} , \quad (15)$$

where $v_E = 30$ km/s is the velocity of the earth relative to the sun, and θ is the angle between the two positions of the earth with vortex on the sun. If the two clocks are making an angle ϕ with the x-axis at time t , then (14) and (15) give:

$$\Delta t = \frac{v_E R}{c^2} [\sin(\phi - \theta) - \sin(\phi)] , \quad (16)$$

the x-axis being given by the straight line between the sun and the earth at time t_0 .

For two clocks, that are fixed on earth ϕ has in general a periodic diurnal variation and θ an annual period. In this particular case, which could be tested, we obtain from (16) a diurnal variation, modulated in amplitude by an annual one. The maximum of the desynchronisation will be attained after 6 months, where the one-way velocity of light measured between the two clocks can change in 12 hours from $c + v_E$ to $c - v_E$, if the link is parallel to the earth velocity. This effect can be distinguished

from temperature delays [17] which share the same period, but can be dephased and do not have the same characteristics. For a link of 1000km, Δt amount 666 ns after 6 months if the link is parallel to the earth velocity. In order to detect it, we need a link and clocks with a relative precision of $4 \cdot 10^{-14}$ over 6 months.

The resolution A4 of the IAU contains the description of various time scales and coordinate systems and for this reason almost already all elements of a correct theoretical description of time on earth, such as realised by a net of atomic clocks. The synchronisation of *TCG* or *TT* cannot be realised by atomic clocks working continuously without resynchronisation, but the synchronisation of *TCB* can be realised, because *BRS* can be considered to be not accelerated respective to the universe. Thus, the metric of a geocentered non-rotating reference frame is non-static and non-stationary, already at order c^{-1} , because of the presence of the terms $g_{0i} = v_{Ei}/c$, which are varying in time. In principle, we should also consider the motion of the whole solar system around the galaxy, and following our reasoning *TCB*-synchronised clocks will slowly desynchronise with a maximum of 5 μs for a link of 1000 km after 250 millions of years, but it can be considered as negligible.

5 Conclusion

It was shown that the natural behaviour of atomic clocks in the vicinity of the earth is not described correctly by the synchronisation of *TCG*. Aside from gravitational effects of the earth, *GRS* corresponds to the application of the principle of equivalence to the earth in free fall around the sun. But in the principle of equivalence the four-dimensionality is important and thus *TCG* can only be realised during a sufficiently small time interval in a three-dimensional neighbourhood of the earth. An extension of *TCG* out of this neighbourhood leads to a non-transitive and not self-consistent clock synchronisation around the earth orbit and to an annual and diurnal desynchronisation of clocks on earth. The author proposes that we adopt the synchronisation of *TCB*, the coordinate time of the solar system as time on earth, which leads to a correct description of the behaviour of clocks on earth. The principle of equivalence is reinterpreted in the light of the works on the conventionality of clock synchronisation in inertial frames: A global and self-consistent definition of time implies that the laws of physics in a locally (four-dimensional) freely falling system are those of the inertial theory and in particular the one-way velocity of light is not invariant on earth. In a sense, the work done here is nothing more than the extension to the solar system of what had already been done by physicists of the timing community: the synchronisation in the rotating frame of the earth is given by the synchronisation in the non-rotating frame. Note finally, that the author does not consider the work done here as definitive. The increasing precision of clocks makes necessary to calculate the one-way transmission time at the order c^{-3} .

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